A phonological approach to implicational relations in inflectional paradigms through gradiently weighted syncretism

In an implicative approach to morphology (Ackerman, Blevins and Malouf 2009) (Henceforth ABM), in which paradigms are viewed as dynamic systems, the contents of an inflectional paradigm cell are predicted through co-reference between cells that represent different morphosyntactic feature combinations (MFC's). Finkel and Stump (2006) show that knowledge of the correct exponent for certain MFCs: 'principal parts' (PPs) can be used to correctly predict other exponents in the paradigm and that for complex paradigms in languages like Ngiti, 3 PPs are needed to make these predictions.

This paper develops a new method that reduces the number of PPs from 3 to 1 in Ngiti and also solves a deficiency in an approach to paradigm completion that was taken by Rosen (2019). PPs can be seen as a kind of base form, whose existence for paradigms is argued for by Albright (2008). In the approach taken here, prediction from a base takes the form of Output-Output Faithfulness (Benua 1997), where weighted constraints in Gradient Symbolic Computation (Smolensky and Goldrick 2016) (GSC) reward the choice of an exponent that matches a PP or base form. (Having the same exponent in two different MFCs is known as syncretism.) Given the existence of inflectional classes, if some MFC is a base, its phonological expression may vary depending on the stem class. The weights of this O-O constraint will vary according to (a) the stem class and (b) the MFC whose exponent is being predicted.

In our approach, a speaker predicts a particular form of a paradigm both through O-O Faithfulness to a base and through input blends of partially-activated exponents occurring both on stems and as manifestations of MFC's (Rosen 2019). The optimal exponent for a stem/MFC combination has the highest Harmony calculated by its activations on the stem and MFC plus any reward from OOBF from being syncretic to the base form.

We demonstrate on paradigm data for Kwerba ((1) below) and Ngiti (not shown for lack of space) from Malouf (2013), both of which exhibit a 'crossing diagonals' pattern which is algebraically impossible to derive solely from gradient inputs (strategy (a) above). The Ngiti paradigm has 16 MFC's and 10 inflectional classes, reflecting the difficulty of the 'paradigm cell-filling problem' described by ABM, in which a speaker must choose the correct form for a given stem and MFC from a complex range of possibilities.

Given an appropriate choice of a base form, the correct output form is chosen as follows. An optimal candidate is the one that achieves highest Harmony as calculated through (a) Faithfulness to gradiently-activated input blends of exponents on both stems and MFC's and (b) Harmonic reward from Faithfulness to the base form whose constraint weight varies among MFC's and among stems. Each MFC and stem representing a stem class is a blend of partially activated exponents. The weights of constraint O-O-BASE-FAITH (OOBF) are represented as two vectors O_M and O_S of dimensions m and s, where m is the number of MFC's and s the number of stem classes.

(1) O-O-BASE-FAITH: "If exponent candidate e_k for MFC m_j matches the base form for the same stem class s_i, add to the Harmony for that candidate the two weights for OOBF for that stem class i and for the MFC j."

(2) Kwerba paradigm (from Malouf 2013)

Cl.	1.aug	1.dim	1.du	1.pl	2.aug	2.dim
1	a	a	ac	ec	a	a
2	a	naN	aN	eN	a	naN
3	a	naN	aN	e	a	naN
4	a	naN	aN	era	a	naN

Cl.	2.du	2.pl	3.aug	3.dim	3.du	3.pl
1	ac	ac	a	a	ac	naN
2	aN	aN	a	naN	aN	naN
3	aN	a	a	naN	aN	a
4	aN	ara	a	naN	aN	ara

One crossing diagonals pattern in Kwerba is shown in red and blue in (2) and in isolation in (3). Predicting the exponents in (3) solely through Faithfulness to gradiently-activated input blends of exponents on stems and MFC's would require contradictory activation hierarchies as shown in (4), where a_{3dim} and a_{s1} represent input activations of exponent /a/ on MFC 3.dim. class 1 stem respectively. The 4 inequalities in (4), which would have to be true to predict the crossing diagonal pattern in (3), can be shown by simple algebraic manipulation to lead to a contradiction. This is equivalent to the 'Xor problem': the simplest unsolvable problem for a linear classifer.

(3)	3.dim.	3.pl.
Stem class 1	a	naN
Stem class 3	naN	a

(4)
$$\begin{aligned} a_{3dim} + a_{s1} &> naN_{3dim} + naN_{s1} \\ naN_{3pl} + naN_{s1} &> a_{3pl} + a_{s1} \\ a_{3dim} + a_{s3} &< naN_{3dim} + naN_{s3} \\ naN_{3pl} + naN_{s3} &< a_{3pl} + a_{s3} \end{aligned}$$

This kind of pattern **can** be predicted when the Harmony for each candidate exponent e_i for each cell in (3) is the sum of its activation on the stem and its activation on the MFC, **plus** the reward for the OOBF constraint if e_i matches the base form 2nd.pl. for that stem (shaded green in (2)) (ac for S_2 and a for S_6.) The OOBF weights are learned values relative to the stem and to the MFC. Those calculations are shown in (5). For S.3 3rd.pl., OOBF has weight 0.06 for stem 3 and 0.07 for MFC 3rd.pl. This boosts the Harmony of /a/, which matches the base form, by 0.13, allowing it to correctly surface instead of /naN/.

This model correctly predicted all exponents for the Kwerba and Ngiti paradigms with activation values of exponents for stems and MFCs and constraint weights for OOBF for each stem and MFC learned from an error-driven algorithm. The algorithm iteratively predicts an exponent for each MFC/stem combination, and, for an incorrect prediction, modifies activations and weights to favour the correct winner.

In addition to reducing implicative complexity, this model's gradient representations naturally model dialectal differences and diachronic changes in paradigms through changes in constraint weights and activations. In further research, we aim to apply this model to other inflectional systems and to represent exponents as phoneme-by-phoneme representations.

(5) Input activations of exponents for stems and MFC's
Each row is a blend of the listed activations on a stem or MFC.
Each column is the contributions to Harmony for that exponent.

Exp.	a	ac	ec	naN	aN	eN	e	era	ara
S_1	.17	.13	.11	.04					
S_3	.13	.		.08	.09		.01		
3.dim.	.14			.26					
3.pl.	.07			.22					.12
OOBF not enough to affect choice of exponent.									
S_1+3.dim.	.31	.13	.11	.30					
$OOBF_{ac}$		.05	$= 0.05_{S1} + 0_{3dim}$						
Total	**.31**	.18	.11	.30					
OOBF not enough to affect choice of exponent.									
S_1+3.pl.	.24	.13	.11	.26					.12
$OOBF_{ac}$		.12	$= 0.05_{S1} + .07_{3pl}$						
Total	.24	.25	.11	**.26**					.12
Zero wt. on OOBF for 3.dim. not enough to boost activation on /a/.									
S_3+3.dim.	.27			.34	.09		.01		
$OOBF_a$	.06	$= .06_{S3} + 0_{3dim}$							
Total	.33			**.34**	.09		.01		
Implied syncretism with [a] on 2.pl. base raises activation on /a/.									
S_3+3.pl.	.20			.30	.09	.01			.12
$OOBF_a$	.13	$= .06_{S3} + .07_{3.pl.}$							
Total	**.33**			.30	.09	.01			.12

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